Love-for-Variety

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Introduction

Love-for-Variety (LV): Utility (productivity) gains from increasing variety of consumer goods (intermediate inputs).

- A natural consequence of the convexity of the utility (production) function.

- Willingness to pay for new goods (inputs); Dixit-Stiglitz (1977), Krugman (1980), Ethier (1982), Romer (1987), etc.

- A central concept in economic growth (Grossman-Helpman 1993; Gancia-Zillibotti 2005, Acemoglu 2008), international trade (Helpman-Krugman 1995), and economic geography (Fujita-Krugman-Venables 1999).

- Though commonly discussed in monopolistic competition settings, also useful in other contexts, such as gains from trade in Armington-type competitive models.

Little known about how LV depends on the underlying demand system outside of **CES with gross substitutes:**

- The LV measure under CES: $\mathcal{L} = 1/(\sigma - 1) > 0,$ where $\sigma > 1$ represents 2 related but distinct concepts,
    - Elasticity of Substitution (ES) across different goods.
    - Price Elasticity (PE) of demand for each good.

  ✓ **One appealing feature:** LV is smaller when ES (and PE) is larger.

  ✓ **One unappealing feature:** LV remains constant, as the variety of available goods increases. Intuitively, many think that LV should decline as the variety increases.

CES is convenient, because knowing PE tells you everything you need to know about ES and LV. But, this is a double-edged sword; It also means that the relation btw PE, ES, and LV are hard-wired and too rigid under CES.

For this reason, some may prefer "Ideal variety approach," but it is less tractable than "Love-for-variety approach."

The Questions: What happens if we drop the restrictive and unrealistic CES assumption?

- How is LV related to the underlying demand structure, such as ES or PE?

Note: ES and PE are distinct concepts outside of CES, which could play different roles shaping LV.

- Under what conditions does LV decline as the variety of available goods increases?

*Departing from CES by introducing empirically plausible **2nd Law of Demand** (PE higher at a higher price) help?*

- Can we develop "Love-for-variety approach" with diminishing LV, which is also tractable?

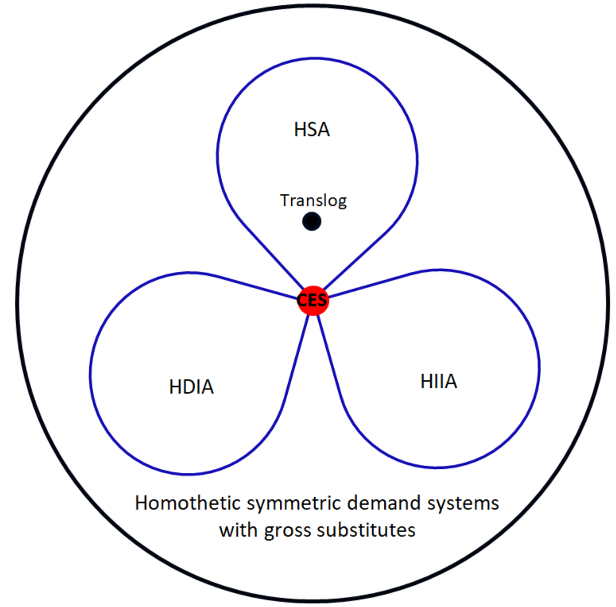
Our Approach to These Questions

- Define **Substitutability,** $\sigma(V)$, & **Love-for-Variety,** $\mathcal{L}(V)$; both depend only on V (the variety of available goods).
    - Under CES, there are independent of V, as $\sigma(V) = \sigma$; $\mathcal{L}(V) = 1/(\sigma - 1)$.
- One's intuition might say:
    - **2nd Law of Demand** implies **Increasing Substitutability**
    - **Increasing Substitutability** implies **Diminishing Love-for-Variety;** $\sigma'(V) > 0 \Longrightarrow \mathcal{L}'(V) < 0$.

- It turns out that this is NOT true under **general symmetric homothetic demand systems.**
  Little can be said about the relations between PE, $\sigma(V)$ & $\mathcal{L}(V)$. "Almost anything goes."

- To capture the above intuition, we need to impose more restrictions. Homotheticity (and symmetry) just too broad.

We turn to the **3 classes of symmetric homothetic demand systems:**
H.S.A. (Homothetic Single Aggregator)
HDIA (Homothetic Direct Implicit Additivity)
HIIA (Homothetic Indirect Implicit Additivity)



- Pairwise disjoint with the sole exception of CES.
- PE can be written as $\zeta_\omega \equiv \zeta(p_\omega/\mathcal{A}(\mathbf{p}))$, where $\mathcal{A}(\mathbf{p})$ is linear homogeneous, a sufficient statistic for the cross-product effects.

Main Results: In each of these 3 classes,

i) $\sigma'(V) \gtreqless 0 \Leftrightarrow \zeta'(p_\omega/\mathcal{A}(\mathbf{p})) \gtreqless 0$.

ii) $\sigma'(V) \gtreqless 0$ for all $V > 0 \Longrightarrow \mathcal{L}'(V) \lesseqgtr 0$ for all $V > 0$.

iii) $\mathcal{L}'(V) = 0$ for all $V > 0 \Leftrightarrow \sigma'(V) = 0$ for all $V > 0$, which occurs iff CES.

Note: Why the converse is not true in general in ii).

$\sigma'(V)$; how the curvature of the marginal utility function changes

$\mathcal{L}'(V)$; how the curvature of the utility function changes.

The monotonicity of the curvature of the marginal utility function implies the monotonicity of the curvature of the utility function, but not the other way around.

The 3 classes offer a tractable way of capturing the intuition that **gains from increasing variety is diminishing, if goods are more substitutable with greater variety of goods.**

Some Remarks Before Proceeding,

- **This paper is all about the *demand side* of LV**.

- **We deliberately make no assumption on the supply side,** to make the results applicable to a wide range of models.

    - **Armington-type competitive trade**, where each differentiated input (or consumer good) is produced and sold by competitive producers, and the variety of available goods, V, changes due to trade liberalization.
    - **Central planning problems**, where the benevolent planner chooses V optimally subject to the innovation cost.
    - **Oligopoly models** with a finite number of oligopolistic firms, some or all of which innovate and produce many different goods.
    - **Monopolistically competitive models**, with a continuum of monopolistically competitive firms innovating and producing zero measure of goods and selling them with positive markups.

- **Neither symmetry nor homotheticity are as restrictive as they look.**

    - By nesting symmetric homothetic demand systems into a upper-tier asymmetric/nonhomothetic demand system, we can create an asymmetric/nonhomothetic demand system.
    - Moreover, one key message is "Almost anything goes," that symmetry/homotheticity restrictions are *not restrictive enough* that we need to look for more restrictions to make further progress.

General Symmetric Homothetic Demand Systems

General Symmetric Homothetic (Input) Demand System

Consider demand system for a continuum of differentiated inputs generated by symmetric CRS production technology.

CRS Production Function	Unit Cost Function
$X(\mathbf{x}) \equiv \min_{\mathbf{p}} \left\{ \mathbf{px} = \int_{\Omega} p_\omega x_\omega d\omega \,\middle|\, P(\mathbf{p}) \geq 1 \right\}$	$P(\mathbf{p}) \equiv \min_{\mathbf{x}} \left\{ \mathbf{px} = \int_{\Omega} p_\omega x_\omega d\omega \,\middle|\, X(\mathbf{x}) \geq 1 \right\}$

$\mathbf{x} = \{x_\omega; \omega \in \overline{\Omega}\}$**:** the input quantity vector; $\mathbf{p} = \{p_\omega; \omega \in \overline{\Omega}\}$**:** the input price vector.

$\overline{\Omega}$, the continuum set of all potential inputs. $\Omega \subset \overline{\Omega}$, the set of available inputs with its mass $V \equiv |\Omega|$.

$\overline{\Omega} \backslash \Omega$: the set of unavailable inputs, $x_\omega = 0$ and $p_\omega = \infty$ for $\omega \in \overline{\Omega} \backslash \Omega$.

Inputs are *inessential*, i.e., $\overline{\Omega} \backslash \Omega \neq \emptyset$ does NOT imply $X(\mathbf{x}) = 0 \Leftrightarrow P(\mathbf{p}) = \infty$.

Duality: Either $X(\mathbf{x})$ or $P(\mathbf{p})$ can be a *primitive*, if linear homogeneity, monotonicity & strict quasi-concavity satisfied

Demand System

Demand Curve (from Shepherd's Lemma)	Inverse Demand Curve
$x_\omega = \dfrac{\partial P(\mathbf{p})}{\partial p_\omega} X(\mathbf{x})$	$p_\omega = P(\mathbf{p}) \dfrac{\partial X(\mathbf{x})}{\partial x_\omega}$

From Euler's Homogenous Function Theorem,

$$\mathbf{px} = \int_{\Omega} p_\omega x_\omega d\omega = \int_{\Omega} p_\omega \frac{\partial P(\mathbf{p})}{\partial p_\omega} X(\mathbf{x}) d\omega = \int_{\Omega} P(\mathbf{p}) \frac{\partial X(\mathbf{x})}{\partial x_\omega} x_\omega d\omega = P(\mathbf{p}) X(\mathbf{x}) = E.$$

The value of inputs is equal to the total value of output under CRS.

Budget Share of $\omega \in \Omega$:	$s_\omega \equiv \dfrac{p_\omega x_\omega}{\mathbf{px}} = \dfrac{p_\omega x_\omega}{P(\mathbf{p})X(\mathbf{x})} = \dfrac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega} \equiv s(p_\omega, \mathbf{p}) = \dfrac{\partial \ln X(\mathbf{x})}{\partial \ln x_\omega} \equiv s^*(x_\omega, \mathbf{x})$

Homogeneity of degree zero → $s_\omega = s(1, \mathbf{p}/p_\omega) = s^*(1, \mathbf{x}/x_\omega)$.
In general, it depends on the whole *distribution* of the prices (quantities) divided by its own price (quantity).

Definition: Gross Substitutability

$$\frac{\partial \ln s(p_\omega; \mathbf{p})}{\partial \ln p_\omega} < 0 \Leftrightarrow \frac{\partial \ln s^*(x_\omega;\ \mathbf{x})}{\partial \ln x_\omega} > 0$$

Price Elasticity of Demand for $\omega \in \Omega$	$\zeta_\omega \equiv -\dfrac{\partial \ln x_\omega}{\partial \ln p_\omega} = \zeta(p_\omega; \mathbf{p}) \equiv 1 - \dfrac{\partial \ln s(p_\omega; \mathbf{p})}{\partial \ln p_\omega} = \zeta^*(x_\omega; \mathbf{x}) \equiv \left[1 - \dfrac{\partial \ln s^*(x_\omega;\ \mathbf{x})}{\partial \ln x_\omega}\right]^{-1} > 1.$

Homogeneity of degree zero implies → $\zeta_\omega = \zeta(1, \mathbf{p}/p_\omega) = \zeta^*(1, \mathbf{x}/x_\omega)$.
In general, it depends on the whole *distribution* of prices (quantities) divided by its own price (quantity).

Definition: The 2nd Law of Demand

$$\frac{\partial \ln \zeta(p_\omega; \mathbf{p})}{\partial \ln p_\omega} > 0 \Leftrightarrow \frac{\partial \ln \zeta^*(x_\omega; \mathbf{x})}{\partial \ln x_\omega} < 0.$$

Clearly, CES does not satisfy the 2nd Law.

Substitutability Measure Across Different Goods

Unit Quantity Vector: $\mathbf{1}_\Omega \equiv \{(1_\Omega)_\omega; \omega \in \overline{\Omega}\}$, where $(1_\Omega)_\omega \equiv \begin{cases} 1 & \text{for} \quad \omega \in \Omega \\ 0 & \text{for} \quad \omega \in \overline{\Omega}\backslash\Omega \end{cases}$

Unit Price Vector: $\mathbf{1}_\Omega^{-1} \equiv \left\{\left(1_\Omega^{-1}\right)_\omega; \omega \in \overline{\Omega}\right\}$, where $\left(1_\Omega^{-1}\right)_\omega \equiv \begin{cases} 1 & \text{for} \quad \omega \in \Omega \\ \infty & \text{for} \quad \omega \in \overline{\Omega}\backslash\Omega \end{cases}$

Note: $\int_\Omega (1_\Omega)_\omega d\omega = \int_\Omega \left(1_\Omega^{-1}\right)_\omega d\omega = |\Omega| \equiv V$.

At the symmetric patterns, $\mathbf{p} = p\mathbf{1}_\Omega^{-1}$ and $\mathbf{x} = x\mathbf{1}_\Omega$,

$$s_\omega = s(1, \mathbf{p}/p_\omega) = s^*(1, \mathbf{x}/x_\omega) = s\left(1, \mathbf{1}_\Omega^{-1}\right) = s^*(1, \mathbf{1}_\Omega) = 1/V$$

$$\zeta_\omega = \zeta(1, \mathbf{p}/p_\omega) = \zeta^*(1, \mathbf{x}/x_\omega) = \zeta\left(1, \mathbf{1}_\Omega^{-1}\right) = \zeta^*(1, \mathbf{1}_\Omega) > 1$$

Clearly, this depends only on V. We propose:

Definition: *The substitutability measure across goods* is defined by

$$\sigma(V) \equiv \zeta\left(1; \mathbf{1}_\Omega^{-1}\right) = \zeta^*(1; \mathbf{1}_\Omega) > 1.$$

We call the case of $\sigma'(V) > (<) 0$ for all $V > 0$, the case of ***increasing (decreasing) substitutability***.

Notes:

- We can also define in terms of Allen-Uzawa Elasticity of Substitution evaluated at the symmetric patterns, which turns out to be equivalent.
- In general, the 2nd Law is neither sufficient nor necessary for increasing substitutability, $\sigma'(V) > 0$.

Love-for-Variety Measure: Commonly defined by the productivity gain from a higher V, holding xV

$$\left.\frac{d\ln X(\mathbf{x})}{d\ln V}\right|_{\mathbf{x}=x\mathbf{1}_\Omega, xV=const.} = \left.\frac{d\ln xX(\mathbf{1}_\Omega)}{d\ln V}\right|_{xV=const.} = \frac{d\ln X(\mathbf{1}_\Omega)}{d\ln V} - 1 > 0$$

Alternatively, LV may be defined by the decline in $P(\mathbf{p})$ from a higher V, at $\mathbf{p} = p\mathbf{1}_\Omega^{-1}$, holding p constant.

$$-\left.\frac{d\ln P(\mathbf{p})}{d\ln V}\right|_{\mathbf{p}=p\mathbf{1}_\Omega^{-1},\ p=const.} = -\frac{d\ln P\left(\mathbf{1}_\Omega^{-1}\right)}{d\ln V} > 0.$$

Both are functions of V only, and equivalent because, by applying $\mathbf{x} = x\mathbf{1}_\Omega$ and $\mathbf{p} = p\mathbf{1}_\Omega^{-1}$ to $\mathbf{px} = P(\mathbf{p})X(\mathbf{x})$,

$$pxV = pP\left(\mathbf{1}_\Omega^{-1}\right)xX(\mathbf{1}_\Omega) \Longrightarrow -\frac{d\ln P\left(\mathbf{1}_\Omega^{-1}\right)}{d\ln V} = \frac{d\ln X(\mathbf{1}_\Omega)}{d\ln V} - 1 > 0.$$

Definition. *The love-for-variety measure* is defined by:

$$\mathcal{L}(V) \equiv -\frac{d\ln P\left(\mathbf{1}_\Omega^{-1}\right)}{d\ln V} = \frac{d\ln X(\mathbf{1}_\Omega)}{d\ln V} - 1 > 0.$$

We call the case of $\mathcal{L}'(V) < (>) 0$ for all $V > 0$, the case of ***diminishing (increasing) love-for-variety***.

Note: $\mathcal{L}(V) > 0$ is guaranteed by the strict quasi-concavity.

Example: Standard CES with Gross Substitutes:

$$X(\mathbf{x}) = Z\left[\int_{\Omega} x_{\omega}^{1-\frac{1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}} \iff P(\mathbf{p}) = \frac{1}{Z}\left[\int_{\Omega} p_{\omega}^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}},$$

where $\sigma > 1$. ($Z > 0$ is TFP or affinity in the preference, in the context of spatial economics)

	CES
Budget Share	$s_{\omega} = \left(\frac{p_{\omega}}{ZP(\mathbf{p})}\right)^{1-\sigma} = \left(\frac{Zx_{\omega}}{X(\mathbf{x})}\right)^{1-1/\sigma}$
Price Elasticity	$\zeta_{\omega} = \sigma > 1$
Substitutability	$\sigma(V) = \sigma > 1$
Love-for-variety	$\mathcal{L}(V) = \frac{1}{\sigma - 1} > 0.$

Under Standard CES,

- PE of demand, $\zeta(p_{\omega}; \mathbf{p}) = \zeta^{*}(x_{\omega}; \mathbf{x})$, is independent of $\mathbf{p}$ or $\mathbf{x}$ and equal to σ.
- Substitutability, $\sigma(V)$, is independent of V and equal to σ.
- LV, $\mathcal{L}(V)$, is independent of V, and equal to a constant, $\mathcal{L}(V) = \mathcal{L} = 1/(\sigma - 1)$, inversely related to σ.

These properties do not hold under general homothetic demand systems.

Example: Generalized CES with Gross Substitutes a la Benassy (1996).

$$X(\mathbf{x}) = Z(V)\left[\int_{\Omega} x_{\omega}^{1-\frac{1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}} \Leftrightarrow P(\mathbf{p}) = \frac{1}{Z(V)}\left[\int_{\Omega} p_{\omega}^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}},$$

Note: $Z(V)$ allows variety to have direct externalities to TFP (or affinity)

	Under Generalized CES
Budget Share	$s_{\omega} = \left(\frac{p_{\omega}}{Z(V)P(\mathbf{p})}\right)^{1-\sigma} = \left(\frac{Z(V)x_{\omega}}{X(\mathbf{x})}\right)^{1-1/\sigma}$
Price Elasticity	$\zeta_{\omega} = \sigma > 1$
Substitutability	$\sigma(V) = \sigma > 1$
Love-for-variety	$\mathcal{L}(V) = \frac{1}{\sigma - 1} + \frac{d \ln Z(V)}{d \ln V}.$

- PE, ζ_{ω}, and Substitutability, $\sigma(V)$, are not affected by $d \ln Z(V)/d \ln V$, "the Benassy residual", which "accounts for" the gap btw LV implied by CES (say, from the markup) & LV implied by productivity growth.
- Benassy (1996) set $d \ln Z(V)/d \ln V = \nu - 1/(\sigma - 1)$, so that $\mathcal{L}(V) = \nu$ is a separate parameter.

Even if you believe in the direct externalities behind the Benassy residual, your estimate of its magnitude depends on the CES structure, which nobody believes.

General Homothetic Demand System: The relation btw $\zeta(p_\omega; \mathbf{p}) = \zeta^*(x_\omega; \mathbf{x})$, $\sigma(V)$, & $\mathcal{L}(V)$ can be complex.

- Whether the 2nd Law holds or not says little about the derivatives of $\sigma(V)$ and $\mathcal{L}(V)$.
- $\sigma(V)$ and $\mathcal{L}(V)$ could be positively related.

(Counter)Example: Weighted Geometric Mean of Symmetric CES:

$$X(\mathbf{x}) \equiv \exp\left[\int_1^\infty \ln X(\mathbf{x};\sigma)\, dF(\sigma)\right], \qquad \text{where} \qquad [X(\mathbf{x};\sigma)]^{1-\frac{1}{\sigma}} \equiv \int_\Omega {x_\omega}^{1-\frac{1}{\sigma}} d\omega$$

and $F(\cdot)$ is a c.d.f. of $\sigma \in (1,\infty)$, satisfying $\int_1^\infty dF(\sigma) = 1$.

	Under Weighted Geometric Mean of CES
Price Elasticity	$\zeta_\omega = \zeta^*(x_\omega;\mathbf{x}) = E_F\left((x_\omega)^{-\frac{1}{\sigma}}/(X(\mathbf{x};\sigma))^{1-\frac{1}{\sigma}}\right)\Big/E_F\left((x_\omega)^{-\frac{1}{\sigma}}/\sigma(X(\mathbf{x};\sigma))^{1-\frac{1}{\sigma}}\right) > 1$
Substitutability	$\sigma(V) = \dfrac{1}{E_F(1/\sigma)} > 1$
Love-for-variety	$\mathcal{L}(V) = E_F\left(\dfrac{1}{\sigma - 1}\right) > 0$

- PE, $\zeta^*(x_\omega;\mathbf{x})$, is not constant, and *violates* the 2nd Law.
- $\sigma(V)$ and $\mathcal{L}(V)$ are both constant, *independent* of V, even though PE is not constant.
- The range of $\sigma(V)$ and $\mathcal{L}(V)$ is $0 < \frac{1}{\sigma(V)-1} \le \mathcal{L}(V) < \infty$, where the equality holds iff F is degenerate.
- Easy to construct a parametric family of cdf's, F, such that $\sigma(V)$ and $\mathcal{L}(V)$ are positively related.

Three Classes of Symmetric Homothetic Demand Systems

However, it is intuitive to think that, as different goods are more substitutable,

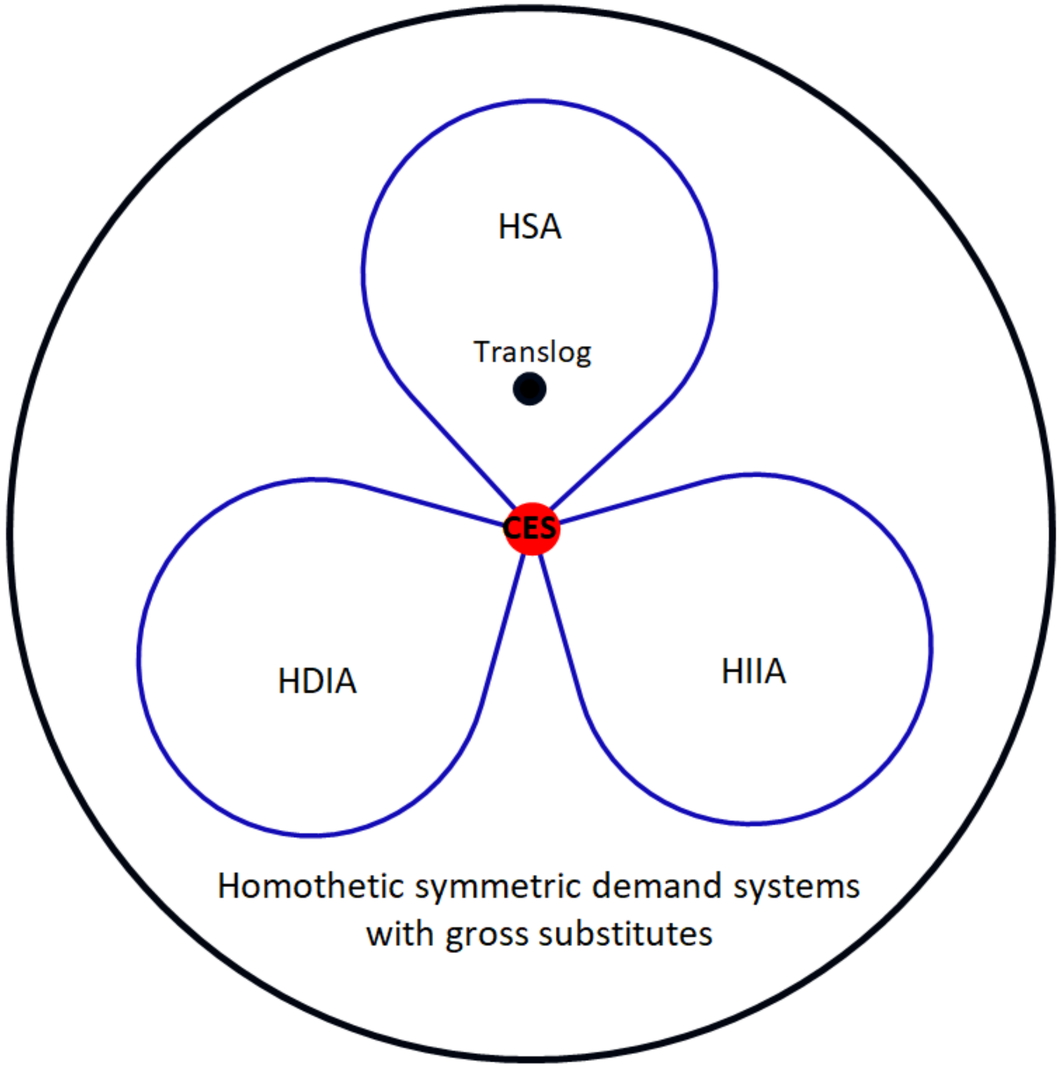
- PE of demand for each good become larger,
- LV becomes smaller.

Homotheticity is too general to capture this intuition!!
It is NOT restrictive enough.

To capture this intuition, we turn to

3 Classes of Symmetric Homothetic Demand Systems:

✓ **Homothetic Single Aggregator (H.S.A.)**
✓ **Homothetic Direct Implicit Additivity (HDIA)**
✓ **Homothetic Indirect Implicit Additivity (HIIA)**



3 Classes of Symmetric Homothetic Demand Systems (with Gross Substitutes & Inessentiality)

$\mathcal{M}[\cdot]$ is a monotone transformation.

Homothetic Direct Implicit Additivity (HDIA):

$$\mathcal{M}\left[\int_{\Omega}^{\square} \phi\left(\frac{Zx_\omega}{X(\mathbf{x})}\right) d\omega\right] \equiv \mathcal{M}\left[\int_{\Omega}^{\square} \phi\left(\frac{x_\omega}{\hat{X}(\mathbf{x})}\right) d\omega\right] \equiv 1$$

$\phi(\cdot): \mathbb{R}_+ \to \mathbb{R}_+$, thus $\hat{X}(\mathbf{x})$, is independent of $Z > 0$, TFP. CES with $\phi(y) = (y)^{1-1/\sigma}, \sigma > 1$.
$\phi(0) = 0; \phi(\infty) = \infty; \phi'(y) > 0 > \phi''(y), 0 < -y\phi''(y)/\phi'(y) < 1$, for $\forall y \in (0, \infty)$.

Homothetic Indirect Implicit Additivity (HIIA):

$$\mathcal{M}\left[\int_{\Omega}^{\square} \theta\left(\frac{p_\omega}{ZP(\mathbf{p})}\right) d\omega\right] \equiv \mathcal{M}\left[\int_{\Omega}^{\square} \theta\left(\frac{p_\omega}{\hat{P}(\mathbf{p})}\right) d\omega\right] \equiv 1$$

$\theta(\cdot): \mathbb{R}_{++} \to \mathbb{R}_+$, thus $\hat{P}(\mathbf{p})$, is independent of $Z > 0$ is TFP. CES with $\theta(z) = (z)^{1-\sigma}, \sigma > 1$.
$\theta(z) > 0, \theta'(z) < 0 < \theta"(z), -z\theta''(z)/\theta'(z) > 1$ for $0 < z < \bar{z} \le \infty$, $\theta(0) = \infty$ and $\theta(z) = 0$ for $z \ge \bar{z}$.

Homothetic Single Aggregator (H.S.A.):

$$s_\omega = \frac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega} = s\left(\frac{p_\omega}{A(\mathbf{p})}\right) \text{ with } \int_{\Omega}^{\square} s\left(\frac{p_\omega}{A(\mathbf{p})}\right) d\omega \equiv 1.$$

$s(\cdot): \mathbb{R}_+ \to \mathbb{R}_+$, thus $A(\mathbf{p})$, is independent of $Z > 0$, TFP. CES with $s(z) = \gamma z^{1-\sigma}, \sigma > 1$.

$s(z) > 0 > s'(z)$ for $0 < z < \bar{z} \le \infty$; $s(z) = 0$ for $z \ge \bar{z}$.

$Z > 0$ shows up when integrating the budget share to obtain $P(\mathbf{p})$ or $X(\mathbf{x})$.

Key Properties of the Three Classes

	Budget Shares: $s_\omega \equiv \dfrac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega} = s(p_\omega; \mathbf{p})$		**Price Elasticity:** $\zeta_\omega \equiv -\dfrac{\partial \ln x_\omega}{\partial \ln p_\omega} = \zeta(p_\omega; \mathbf{p})$
CES	$s_\omega = \left(\dfrac{p_\omega}{ZP(\mathbf{p})}\right)^{1-\sigma}$		σ
H.S.A. $s(\cdot)$	$s_\omega = s\left(\dfrac{p_\omega}{A(\mathbf{p})}\right)$	$\frac{P(\mathbf{p})}{A(\mathbf{p})} \neq c$, unless CES	$\zeta^S\left(\dfrac{p_\omega}{A(\mathbf{p})}\right)$; $\zeta^S(z) \equiv 1 - \dfrac{zs'(z)}{s(z)} > 1$
HDIA $\phi(\cdot)$	$s_\omega = \dfrac{p_\omega}{P(\mathbf{p})}(\phi')^{-1}\left(\dfrac{p_\omega}{B(\mathbf{p})}\right)$	$\frac{P(\mathbf{p})}{B(\mathbf{p})} \neq c$, unless CES	$\zeta^D\left((\phi')^{-1}\left(\dfrac{p_\omega}{B(\mathbf{p})}\right)\right)$; $\zeta^D(y) \equiv -\dfrac{\phi'(y)}{y\phi''(y)} > 1$
HIIA $\theta(\cdot)$	$s_\omega = \dfrac{p_\omega}{C(\mathbf{p})}\theta'\left(\dfrac{p_\omega}{P(\mathbf{p})}\right)$	$\frac{P(\mathbf{p})}{C(\mathbf{p})} \neq c$, unless CES	$\zeta^I\left(\dfrac{p_\omega}{\hat{P}(\mathbf{p})}\right)$; $\zeta^I(z) \equiv -\dfrac{z\theta''(z)}{\theta'(z)} > 1.$

$A(\mathbf{p}), B(\mathbf{p}), C(\mathbf{p})$: each defined implicitly by the adding-up constraint, $\int_\Omega s_\omega d\omega \equiv 1$. Clearly, they are all linear homogenous.

We focus on these three classes for two reasons.

- They are pairwise disjoint with the sole exception of CES.
- PE $= \zeta_\omega \equiv \zeta\left(\frac{p_\omega}{\mathcal{A}(\mathbf{p})}\right)$, where $\mathcal{A}(\mathbf{p})$ is linear homogenous, a sufficient statistic, capturing all the cross-product effects.

Key Properties of the Three Classes, Continued.

	Price Elasticity: $\zeta(p_\omega;\mathbf{p})$	Substitutability : $\sigma(V)$	Love-for-Variety: $\mathcal{L}(V)$
H.S.A.	$\zeta_\omega = \zeta\left(\frac{p_\omega}{A(\mathbf{p})}\right)$	$\sigma(V) = \zeta^S\left(s^{-1}\left(\frac{1}{V}\right)\right)$	$\mathcal{L}(V) = \frac{1}{\mathcal{E}_H\left(s^{-1}(1/V)\right)},$

where $\zeta^S(z) \equiv -\frac{zH''(z)}{H'(z)} > 1$ and $\mathcal{E}_H(z) \equiv -\frac{zH'(z)}{H(z)} > 0$, with $H(z) \equiv \int_z^{\bar{z}} \frac{s(\xi)}{\xi} d\xi > 0$.

HDIA	$\zeta_\omega = \zeta^D\left((\phi')^{-1}\left(\frac{p_\omega}{B(\mathbf{p})}\right)\right)$	$\sigma(V) = \zeta^D\left(\phi^{-1}\left(\frac{1}{V}\right)\right)$	$\mathcal{L}(V) = \frac{1}{\mathcal{E}_\phi(\phi^{-1}(1/V))} - 1$

where $\zeta^D(y) \equiv -\frac{\phi'(y)}{y\phi''(y)} > 1$ and $0 < \mathcal{E}_\phi(y) \equiv \frac{y\phi'(y)}{\phi(y)} < 1$.

HIIA	$\zeta_\omega = \zeta^I\left(\frac{p_\omega}{\hat{P}(\mathbf{p})}\right)$	$\sigma(V) = \zeta^I\left(\theta^{-1}\left(\frac{1}{V}\right)\right)$	$\mathcal{L}(V) = \frac{1}{\mathcal{E}_\theta\left(\theta^{-1}(1/V)\right)}$

where $\zeta^I(z) \equiv -\frac{z\theta''(z)}{\theta'(z)} > 1$ and $\mathcal{E}_\theta(z) \equiv -\frac{z\theta'(z)}{\theta(z)} > 0$.

Note: In all three classes, $\mathcal{L}(V)$ depends on the curvature of a function of a single variable, $H(\cdot), \phi(\cdot), \theta(\cdot)$, while $\sigma(V)$ depends on the curvature of its derivative. $H'(\cdot), \phi'(\cdot), \theta'(\cdot)$.

Theorem: Under H.S.A., HDIA, and HIIA,

- $\sigma'(V) > 0$ iff the 2nd law holds.
- $\sigma'(V) \gtreqless 0$ for all $V \in (V_0, \infty) \Longrightarrow \mathcal{L}'(V) \lesseqgtr 0$ for all $V \in (V_0, \infty)$.

The converse is not true in general. However,

- $\mathcal{L}'(V) = 0$ for all $V \in (V_0, \infty) \Leftrightarrow \sigma'(V) = 0$ for all $V \in (V_0, \infty)$.

In particular,

- $\mathcal{L}'(V) = 0$ for all $V > 0 \Leftrightarrow \sigma'(V) = 0$ for all $V > 0 \Leftrightarrow CES$.

The 2nd Law

$\zeta(p_\omega; \mathbf{p})$ is increasing in p_ω

$\zeta^*(x_\omega; \mathbf{x})$ is decreasing in x_ω

$\Updownarrow$

Increasing Substitutability

$\sigma'(V) > 0$ for all $V > 0$.

$\Longrightarrow$

Diminishing Love-for-Variety

$\mathcal{L}'(V) < 0$ for all $V > 0$.

Concluding Remarks

What We Did in This Paper

- Under CES: Constant PE, Constant ES, and Constant LV, with the tight relation between them.
- We asked how departing from CES to allow for the 2nd Law in PE affects ES and LV.
- We defined **Substitutability & Love-for-Variety,** both depend only on the variety of available goods under General Homothetic Symmetric Demand Systems
- **Intuition:** 2nd Law of Demand ⟹ Increasing Substitutability ⟹ Diminishing Love-for-Variety
  - In general, Not True!!
  - Under 3 classes (H.S.A., HDIA, and HIIA), True!!
    - because, in all three, PE for each good is a function of its own price divided by a single aggregator of all prices of available goods.

Potential Applications with Our Conjectures

- **Armington Trade Models:** In all 3 classes, country size matters under the 2nd Law. In larger countries, which already have access to larger variety of goods, high trade elasticity ⟹ Smaller gains from trade.
- **Static Monopolistic Competition:** In all 3 classes, The 2nd Law ⟺ Procompetitive Entry ⟹ Excessive Entry.
- **Romer-type Endogenous Growth with Expanding Variety.**
  - Under CES, too little innovation in the equilibrium.
  - Under Three Classes with the 2nd law, equilibrium innovation can be too little or too much.